Impulsive motion of an infinite plate in a compressible fluid with non-uniform external flow

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The impulsive movement of a plate (Rayleigh problem) is considered for a compressible flow in which, prior to initiation of the motion, both velocity and enthalpy gradients exist normal to the plate. The solution is valid for large times, and the external gradients are chosen of such magnitude that their effects enter to the same order as displacement effects due to induced vertical velocity. This displacement effect is influenced by the external enthalpy gradient. Both insulated wall and a step change in wall enthalpy are considered. For the insulated wall it is found that the part of the displacement solution which is uninfluenced by the external gradient requires a term in the logarithm of Reynolds number (based on time). This differs in principle from the case for a constant wall enthalpy. Displacement with uniform outside flow affects heat transfer but not skin friction, just the opposite from the corresponding results for the steady two-dimensional semi-infinite flat plate. The influence of external gradients on skin friction, heat transfer and adiabatic wall enthalpy is given.

1. Introduction

Boundary-layer flows in which the flow external to the boundary layer is rotational have been studied extensively in recent years, both at the stagnation point and for the flat plate. The impetus for these studies has come both from practical considerations of hypersonic flow about blunt bodies with curved, detached shocks, and from the fact that the problem appears at first to be much simpler than it actually is. A comprehensive study of vorticity interaction as one of several second-order effects for incompressible flow including a perspective of the quite controversial literature can be found in Van Dyke (1960), which emphasizes the necessity of accounting properly for the mutual interaction between inside and outside flows.

External vorticity generated by a hypersonic curved shock implies entropy, and therefore also temperature, gradients normal to the body surface. Near a blunt nose, where the local Mach number is small, this temperature variation is not important. However, there can exist regions further downstream where an external temperature gradient is important for boundary-layer development. On a cylindrical or conical afterbody, locations can be found where the external temperature gradient is as large, relative to the thermal boundary-layer temperature gradient, as the external velocity gradient relative to the momentum boundary-layer velocity gradient. Since on the after-body the streamwise pressure gradient may be relatively small, this focuses attention on the compressible flat-plate boundary layer with non-vanishing shear and heat transfer in the outer flow.

A number of studies of compressible vorticity interaction either ignore or fail to account properly for the interaction of inside and outside flow (Li 1955; Rogers 1961; Ferri, Zakkay & Ting 1961; and Ovchinnikov 1960). A matched inner and outer expansion method for compressible flow is described by Van Dyke (1961) in a generalization of his incompressible analysis with application to the nose of a blunt body. Maslen (1962) discusses the same problem and presents solutions for stagnation point, cylinder and flat plate. The procedure, however, becomes quite complicated in general and even in the case of the flat plate recourse to numerical integration becomes necessary.

In the present study, the effects of external velocity and temperature gradients in a viscous, compressible fluid are considered for the much simpler but related problem of the infinite, impulsively moved flat plate (Rayleigh problem). The Rayleigh problem in compressible flow resembles boundary-layer flow more closely than in incompressible flow since a velocity component normal to the plate exists through density variations. Thus coupling between inner and outer flow can exist. For the present problem, it is found possible to integrate readily the equations and thus to easily explore various features of the interaction, provided suitable restrictions are placed on the magnitude of the external gradients.

The impulsively moved compressible Rayleigh plate problem with uniform outside flow has been extensively investigated using two alternate approaches. First, the problem has been linearized with the assumption of small Mach number. This approach was followed by Howarth (1951) and later Hanin (1960) who obtained solutions for both large and small values of time elapsed from initiation of the motion. This method is not of interest here due to its small Mach-number restriction. Alternately, solutions restricted to large time only have been obtained by following a 'boundary-layer' approach treating separately and then matching inner and outer flows. This approach was taken by Van Dyke (1952) and later Stewartson (1955). The work of Stewartson has recently been generalized by Li (1960) to include the effect of surface mass transfer. All these studies consider only the insulated wall. These large-time solutions have an analogue in the leading-edge shock-wave boundary-layer interaction on a semi-infinite flat plate, and just as in that problem, it is important to distinguish here between strong a weak interaction. In weak interaction theory the interaction only modifies slightly a basic flow, and the time considered is larger than that for strong interaction. The work of Van Dyke (1952) deals with weak interaction, that of Stewartson (1955) and Li (1960) with strong interaction.

Since in the present study the effects of the external gradients are taken to be small, the approach of Van Dyke (1952) is followed throughout, modified to include external gradients and wall heat transfer with constant wall temperature. It is found that the case of the insulated wall and the constant-wall-temperature case are basically different, in that a term in the logarithm of the Reynolds number appears in the insulated wall problem but not the other. Further, the solution to the former case cannot be completely determined. The present insulated results differ considerably from those of Van Dyke (1952) through the omission there of certain critical terms.†

2. Definition of problem

A flat plate of infinite extent in x moves in its own plane (see figure 1). This motion accelerates fluid in the x-direction to velocity u at distance y measured normal to the plate. The density ρ varies through heating by viscous dissipation and this induces a velocity v perpendicular to the plate. The motion depends on time t, having been initiated at $t = 0$ by impulsive acceleration of the plate from rest to constant velocity U_0.

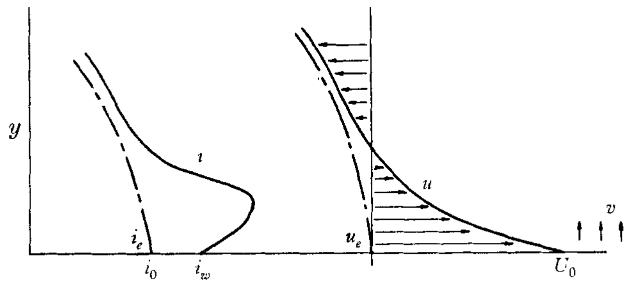

FIGURE 1. Enthalpy and velocity profiles; —·—, initial steady values.

Prior to initiation of the motion, a y-dependent but steady enthalpy i_e and velocity u_e parallel to the plate exist.‡ At the surface of the plate, the enthalpy i is constant at i_0 as long as the plate is at rest. After initiation of the motion, the surface enthalpy becomes i_w, considered for the present to be constant.

With viscosity μ and pressure p, and assuming a Prandtl number of unity, the equations of motion are

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial y}(\rho v) = 0, \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right), \tag{2}$$

$$\rho\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y}+\frac{4}{3}\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right), \tag{3}$$

$$\rho\left(\frac{\partial i}{\partial t}+v\frac{\partial i}{\partial y}\right) = \frac{\partial p}{\partial t}+v\frac{\partial p}{\partial y}+\frac{\partial}{\partial y}\left(\mu\frac{\partial i}{\partial y}\right)+\mu\left[\left(\frac{\partial u}{\partial y}\right)^2+\frac{4}{3}\left(\frac{\partial v}{\partial y}\right)^2\right]. \tag{4}$$

Following Van Dyke (1952) the perpendicular co-ordinate y is transformed by a Howarth transformation. In terms of new variables

$$Y = \int_0^y \frac{\rho}{\rho_0}dy, \quad T = t,$$

† The suggestion that a logarithmic term in Reynolds number becomes necessary when the missing terms are restored is due to Professor Van Dyke.

‡ In what follows, subscripts are taken to have the following meanings: aw, adiabatic wall conditions; e, conditions prior to initiation of the motion; w, conditions at the surface of the plate after acceleration to the constant velocity U_0; 0, conditions at the surface of the plate prior to initiation of motion.

and assuming μ to be proportional to i and the specific heat to be constant, the equations of motion can be written as

$$\frac{1}{\rho_0}\frac{\partial p}{\partial Y} = \tfrac{4}{3}\nu_0 \frac{\partial}{\partial Y}\left(\frac{p}{p_0}\frac{\partial v}{\partial Y}\right) - \frac{\partial v}{\partial T}, \tag{5}$$

$$\frac{\partial u}{\partial T} - \nu_0 \frac{\partial^2 u}{\partial Y^2} = \nu_0 \frac{\partial}{\partial Y}\left(\frac{p-p_0}{p_0}\frac{\partial u}{\partial Y}\right), \tag{6}$$

$$\frac{\partial i}{\partial T} - \nu_0\frac{\partial^2 i}{\partial Y^2} - \nu_0\left(\frac{\partial u}{\partial Y}\right)^2 = \nu_0 \frac{p-p_0}{p_0}\left(\frac{\partial u}{\partial Y}\right)^2 + \tfrac{4}{3}\nu_0\frac{p}{p_0}\left(\frac{\partial v}{\partial Y}\right)^2 + \frac{1}{\rho}\frac{\partial p}{\partial T} + \nu_0\frac{\partial}{\partial Y}\left(\frac{p-p_0}{p_0}\frac{\partial i}{\partial Y}\right), \tag{7}$$

where ν_0 is the kinematic viscosity.

When the density is constant and also when the flow is steady in time there is no vertical velocity, as can be seen from (1). Then from (5) the pressure is constant, and the right side of both (6) and (7) vanishes. In general, however, this is not true. But if (5), (6) and (7) are made dimensionless with $(\nu_0 T)^{\frac{1}{2}}$ for the vertical co-ordinate, and $(\nu_0/T)^{\frac{1}{2}}$ for the vertical velocity, it is found that the ratio of right to left sides in all three is $\nu_0/U_0^2 T$, the reciprocal of a Reynolds number based on time. Thus for sufficiently large times the general problem reduces to the incompressible one through proper transformation of co-ordinates, as is well known in boundary-layer theory.

For times that are large, but not so large that the right sides can be completely neglected, an approximation scheme exists which makes use of the boundary-layer property that viscous effects are confined to the neighbourhood of the plate, and that a considerable extent of fluid exists further out where the effects of viscosity are not important. Appropriate solutions are found both in the inner and outer regions which are then matched. This procedure will also be followed here. We consider at first the first approximation to the outer flow.

In the present problem, the outer flow is non-uniform. Since it is steady and without vertical velocity, one finds from (6)

$$\partial^2 u_e/\partial Y^2 = 0 \quad u_e = \omega Y \quad (\text{satisfying } u_e = 0 \text{ at } Y = 0).$$

Equation (7) becomes
$$\frac{\partial^2 i_e}{\partial Y^2} + \left(\frac{\partial u_e}{\partial Y}\right)^2 = 0.$$

The outer flow velocities are now taken to be so small that dissipative heating in the outer flow is not important. Then

$$i_e = i_0 + \bar{\omega} Y \quad (\text{satisfying } i_e = i_0 \text{ at } Y = 0).$$

The stretched co-ordinate Y is related to the physical co-ordinate through this initial enthalpy profile.

It is important to note that this outer flow is 'inviscid' only in the sense that viscous and heat-conduction terms in the equations of motion vanish. It is not necessary to postulate, as has sometimes been done, an outer flow that is rotational but nevertheless somehow inviscid (because μ and k, the thermal conductivity, vanish outside the boundary layer!).

3. First approximation to inner flow

Suppose first the time to be so large that the right sides of (5), (6) and (7) can be neglected. Since (6) is then linear, the velocity distribution can be immediately written as

$$u/U_0 = \operatorname{erfc}\eta + (\omega Y/U_0), \quad \text{where} \quad \eta = \tfrac{1}{2}Y/(\nu_0 T)^{\frac{1}{2}}. \tag{8}$$

With the velocity known, (7) also becomes linear and the external enthalpy simply can be added on. However, use is made of the Crocco integral, and this requires additional modifications to allow for the outside flow.

Suppose there exists a generalized Crocco integral

$$i = A(Y) + Bu - \tfrac{1}{2}u^2, \tag{9}$$

where B is a constant but A is a function of Y. If

$$A = i_e + \tfrac{1}{2}u_e^2 - (u_e/U_0)(i_w - i_0 + \tfrac{1}{2}U_0^2) \quad \text{and} \quad B = (i_w - i_0 + \tfrac{1}{2}U_0^2)/U_0,$$

then it can be verified that (7) is satisfied, as long as u_e is linear in Y. Thus even with non-uniform outside flow a Crocco integral exists. Dropping the u_e^2 term (slow outer flow), and defining plate Mach number M in terms of the initial velocity of sound at the plate surface, (9) can then be written, using (8), as

$$\begin{aligned}(i/i_0) = 1 + \{(i_w/i_0) - 1 + \tfrac{1}{2}(\gamma-1)M^2\}\operatorname{erfc}\eta - \tfrac{1}{2}(\gamma-1)M^2(\operatorname{erfc}\eta)^2 \\ - (\omega Y/U_0)(\gamma-1)M^2\operatorname{erfc}\eta + (\varpi Y/i_0),\end{aligned} \tag{10}$$

where γ is the ratio of the specific heats.

These results are not quite as simple as they appear since they are expressed in terms of the stretched co-ordinate Y. To transform back, one uses, due to constant pressure in this approximation,

$$\partial y/\partial Y = \rho_0/\rho = i/i_0.$$

From (10),

$$\begin{aligned} y &= \int_0^Y \frac{i}{i_0}\,dY \\ &= Y\Bigg\{1 + \left(\frac{i_w}{i_0} - 1 + \frac{\gamma-1}{2}M^2\right)\left[\operatorname{erfc}\eta + \frac{1}{\sqrt{\pi}}\frac{1}{\eta}(1-e^{-\eta^2})\right] \\ &\quad - \frac{\gamma-1}{2}M^2\left[(\operatorname{erfc}\eta)^2 + \frac{1}{\sqrt{\pi}}\frac{2}{\eta}(1-e^{-\eta^2}\operatorname{erfc}\eta) - \sqrt{\frac{2}{\pi}}\frac{1}{\eta}\operatorname{erf}\sqrt{2}\,\eta\right] \\ &\quad - \frac{\omega Y}{U_0}\frac{\gamma-1}{2}M^2\left[\operatorname{erfc}\eta - \frac{1}{\sqrt{\pi}}\frac{1}{\eta}e^{-\eta^2} + \frac{1}{2\eta^2}\operatorname{erf}\eta\right] + \frac{\varpi Y}{2i_0}\Bigg\}. \end{aligned} \tag{11}$$

Thus both the external velocity and enthalpy gradients appear in a complex way in the velocity profile when considered in the physical co-ordinate y, although no explicit enthalpy dependence is shown by (8). The external velocity gradient appears in the equations coupled to the square of the Mach number; the external enthalpy gradient does not. It may be noted that, although the external profiles are differently expressed in y or Y, the external gradients at the wall are the same expressed in either co-ordinate.

It is appropriate at this point to consider in what form these gradients enter the problem and to make more precise what is meant by their being small. Both gradients appear as $\omega Y/U_0$ and $\overline{\omega} Y/i_0$. Both factors are taken to be small compared to unity. Consideration of $\overline{\omega}$ will be left for § 4; for the present we write

$$\omega Y/U_0 = 2\eta\omega T(\nu_0/U_0^2 T)^{\frac{1}{2}}.$$

A typical value for 2η near the middle of the boundary layer will be of order unity. The restriction to large times and small gradients will relate $\omega Y/U_0$ to $(\nu_0/U_0^2 T)^{\frac{1}{2}}$ through a particular choice of the product ωT. If this is arbitrarily taken to be of order unity, then in the centre of the boundary layer $\omega Y/U_0$ will be of the same order as $(\nu_0/U_0^2 T)^{\frac{1}{2}}$. Now it is well known that effects due to the vertical velocity, which have been neglected so far, enter as $(\nu_0/U_0^2 T)^{\frac{1}{2}}$. The vertical velocity itself can be calculated from (11) and the property of the co-ordinate transformation that $v = \partial y/\partial T$. There results

$$\begin{aligned}\frac{v}{U_0} = \left(\frac{\nu_0}{\pi U_0^2 T}\right)^{\frac{1}{2}} \Bigg\{ \left(\frac{i_w}{i_0} - 1 + \frac{\gamma-1}{2} M^2\right)(1-e^{-\eta^2}) \\ -(\gamma-1) M^2\left(1 - e^{-\eta^2}\operatorname{erfc}\eta - \frac{\operatorname{erf}\sqrt{2}\,\eta}{\sqrt{2}}\right)\Bigg\} \\ + \frac{4\nu_0\omega}{U_0^2}\frac{\gamma-1}{2} M^2\left(\frac{1}{\sqrt{\pi}}\eta\, e^{-\eta^2} - \tfrac{1}{2}\operatorname{erf}\eta\right).\end{aligned} \qquad (12)$$

For the particular choice of ωT made, it is clear that the effects of external gradients enter to the same order as displacement effects induced by this vertical velocity and therefore that these must also be taken into account in a consistent theory. This is done in the following section by first finding the effect of streamline displacement on the outer flow. The resulting pressure is used thereafter, together with the first approximations just given above for u, v and i, to evaluate the neglected right sides in (6) and (7) and to solve the resulting inhomogeneous equations for second-approximation corrections to u, v and i.

4. Outer flow

In the outer portions of the fluid not yet reached by the diffusing viscous zone there is, nevertheless, an effect of the plate's motion due to the vertical velocity calculated previously. The situation is equivalent to that of motion established by a piston moving vertically with velocity

$$v = (\nu_0/\pi T)^{\frac{1}{2}}\,[(i_w/i_0) - 1 + \tfrac{1}{2}(\gamma-1)M^2(\sqrt{2}-1)] - (\nu_0\omega/U_0)\,(\gamma-1)\,M^2 \qquad (13)$$

(obtained from (12) in the limit of large η).

This motion forms a small disturbance which will be calculated by linearization about the non-uniform external flow. In the outer flow the viscous terms in the equations of motion can be shown to be of order $M^2(\nu_0/U_0^2T)$ compared to the others; the disturbance is therefore considered to be inviscid.

In equations (1) to (4), without viscous terms, put

$$\rho = \rho_e + \tilde{\rho}, \quad u = u_e + \tilde{u}, \quad v = \tilde{v}, \quad p = p_0 + \tilde{p}, \quad i = i_e + \tilde{\imath}.$$

Here ρ_e, u_e and i_e depend on y but not on t. There results

$$\frac{\partial \tilde{\rho}}{\partial t} + \frac{\partial}{\partial y}(\rho_e \tilde{v}) = 0, \quad \rho_e\left(\frac{\partial \tilde{u}}{\partial t} + \tilde{v}\frac{\partial u_e}{\partial y}\right) = 0,$$

$$\frac{\rho_e \partial \tilde{v}}{\partial t} + \frac{\partial \tilde{p}}{\partial y} = 0, \quad \rho_e\left(\frac{\partial \tilde{\imath}}{\partial t} + \tilde{v}\frac{\partial i_e}{\partial y}\right) - \frac{\partial \tilde{p}}{\partial t} = 0.$$

From these equations it is possible to obtain after some manipulation

$$\frac{\partial^2 \tilde{v}}{\partial t^2} - a_e^2 \frac{\partial^2 \tilde{v}}{\partial y^2} = 0,$$

where a_e is the velocity of sound in the initial flow. Thus an acoustic equation results for $\tilde{v}$, with a variable velocity of propagation arising from the non-uniform outside enthalpy. The non-uniform outside velocity does not affect this phase of the problem.

Recalling that i_e is linear in the transformed variable Y, whereas the present considerations refer to the physical co-ordinate y, there results for the perturbation velocity the equation

$$\frac{\partial^2 \tilde{v}}{\partial t^2} - a_0^2\left(1 + \frac{2\overline{\omega} y}{i_0}\right)^{\frac{1}{2}} \frac{\partial^2 \tilde{v}}{\partial y^2} = 0, \tag{14}$$

where a_0 is the velocity of sound at the plate.

Introducing the transformations

$$1 + \frac{2\overline{\omega} y}{i_0} = \left(1 + \frac{3}{2}\frac{\overline{\omega}\beta}{i_0}\right)^{\frac{4}{3}}, \quad \tilde{v} = \left(1 + \frac{3}{2}\frac{\overline{\omega}\beta}{i_0}\right)^{\frac{1}{3}}\phi,$$

equation (14) becomes

$$\frac{\partial^2 \phi}{\partial t^2} - a_0^2\left[\frac{\partial^2 \phi}{\partial \beta^2} - \frac{7}{16}\frac{\overline{\omega}^2}{i_0^2}\frac{\phi}{(1 + \frac{3}{2}\overline{\omega}\beta/i_0)^2}\right] = 0. \tag{15}$$

The last term on the left of (15) represents dispersion of a travelling wave which can be thought of as due to reflexions arising in the non-uniform medium. With *a posteriori* justification for a sufficiently small external enthalpy gradient, this term will be neglected.

Then a solution of (15) is an outgoing wave of arbitrary wave form

$$\phi = f\{t - (\beta/a_0)\},$$

which after transforming back and dropping terms of order $(\overline{\omega} y/i_0)^2$ gives, as a solution to (14),

$$\tilde{v} = \left(1 + \frac{\overline{\omega} y}{4 i_0}\right) f\left\{t - a_0^{-1} y\left(1 - \frac{\overline{\omega} y}{4 i_0}\right)\right\}.$$

This wave is travelling, unmodified in shape, along a curved ray with changing amplitude. The function f is determined at $y = 0$ from (13). There results

$$\tilde{v} = \left(1+\frac{\overline{\omega}y}{4i_0}\right)\left\{\left[\frac{i_w}{i_0}-1+\frac{\gamma-1}{2}M^2(\sqrt{2}-1)\right]\right.$$
$$\left.\times\left\{\nu_0\Big/\pi\left[t-a_0^{-1}y\left(1-\frac{\overline{\omega}y}{4i_0}\right)\right]\right\}^{\frac{1}{2}}-\frac{\nu_0\omega}{U_0}(\gamma-1)M^2\right\}. \quad (16)$$

Once f has been determined, it is found after some manipulation that the ratio of the neglected term in (15) to the second term on the left is, neglecting terms of order $(\overline{\omega}y/i_0)^2$ and ν_0/U_0^2t,

$$R = \frac{7}{12}\left(\frac{\overline{\omega}a_0t}{i_0}\right)^2\left[1-\frac{y}{a_0t}\right]^2.$$

Thus dispersion and resulting distortion in wave shape assume importance only in regions away from the wave-front, but even for the largest departure, $y = 0$, the effect is negligible if $(\overline{\omega}a_0t/i_0)^2$ is sufficiently small. $(\overline{\omega}a_0t/i_0)^2 \ll 1$ therefore will be taken as a measure of the smallness of the external enthalpy gradient, so far undefined. Obviously, if $(\overline{\omega}a_0t/i_0)^2$ is small, then $(\overline{\omega}y/i_0)^2$ is even smaller.

The combined limitations then lead to the dual restrictions that

$$\nu_0^2/U_0^4 \ll t^2 \ll i_0^2/a_0^2\overline{\omega}^2, \quad \omega t = O(1).$$

As an interesting aside, it may be mentioned that an external enthalpy varying as the fourth power of y leads rigorously to distortionless propagation.

Once $\tilde{v}$ is known, $\tilde{p}$ may be obtained from the relation

$$\partial\tilde{p}/\partial t = -\gamma p_0 \partial\tilde{v}/\partial y.$$

There results

$$\frac{\tilde{p}}{p_0} = \left(1-\frac{\overline{\omega}a_0t}{2i_0}+\frac{\overline{\omega}y}{4i_0}\right)\left[\frac{i_w}{i_0}-1+\frac{\gamma-1}{2}M^2(\sqrt{2}-1)\right]\gamma M\left\{\nu_0\Big/\pi U_0^2\left[t-a_0^{-1}y\left(1-\frac{\overline{\omega}y}{4i_0}\right)\right]\right\}^{\frac{1}{2}}$$
$$-\frac{\nu_0\omega}{U_0^2}\gamma(\gamma-1)M^3\left(1-\frac{\overline{\omega}a_0t}{4i_0}-\frac{\overline{\omega}y}{4i_0}\right). \quad (17)$$

This is the second approximation to the pressure in the outer flow. It contains contributions from both the external velocity and enthalpy gradients. Since this pressure is ultimately to be used in the approximate evaluation of the right-hand terms in (6) and (7) in the boundary layer, the pressure in the boundary layer is needed. It turns out that since the inhomogeneous terms are needed only to order $(\nu_0/U_0^2T)^{\frac{1}{2}}$, the pressure in the boundary layer is required correct only to this order and is simply

$$\frac{p-p_0}{p_0} = \left[\frac{i_w}{i_0}-1+\frac{\gamma-1}{2}M^2(\sqrt{2}-1)\right]\gamma M\left(\frac{\nu_0}{\pi U_0^2T}\right)^{\frac{1}{2}}\left(1-\frac{\overline{\omega}a_0T}{2i_0}\right). \quad (18)$$

Thus, to the order carried in this analysis and for the given restrictions on time and external gradients, the induced pressure in the boundary layer consists of two terms, only one of which contains the external enthalpy gradient. This may be called the vorticity-induced pressure. The other term agrees with that obtained by Van Dyke (1952) and may be called the conventional displacement induced pressure.

5. Second approximation to inner flow

The correction to the velocity (equation (8)) and enthalpy (equation (10)) due to boundary-layer displacement is now obtained as follows.

Let $u = u_1 + \Delta u$, where u_1 is given by (8), and Δu is a small correction. Substitute this into (6), with the term on the right calculated from u_1 and the pressure from (18). This leads to an inhomogeneous heat-conduction equation for Δu whose solution is found to be

$$\frac{\Delta u}{U_0} = \left[\frac{i_w}{i_0} - 1 + \frac{\gamma-1}{2} M^2(\sqrt{2}-1)\right] \frac{2\gamma M}{\sqrt{\pi}} \left(\frac{\nu_0}{\pi U_0^2 T}\right)^{\frac{1}{2}} \left(1 - \frac{\overline{\omega} a_0 T}{6 i_0}\right) \eta\, e^{-\eta^2}. \tag{19}$$

Similarly, let $i = i_1 + \Delta i$, where i_1 is given by (10). Following the same procedure with (7), with the term on the right calculated from u_1, i_1, the pressure from (18), v from (12), ρ from $\rho = \rho_0 i_1/i_0$, and Δu from (19), leads to an inhomogeneous heat-conduction equation for Δi whose solution eventually is found to be

$$\begin{aligned}\frac{\Delta i}{i_0} = {} & \left[\frac{i_w}{i_0} - 1 + \frac{\gamma-1}{2} M^2(\sqrt{2}-1)\right](\gamma-1) M \left(\frac{\nu_0}{\pi U_0^2 T}\right)^{\frac{1}{2}} \\ & \times \left\{\left[1 - \frac{i_w}{i_0} e^{-\eta^2} + \left(\frac{i_w}{i_0} - 1 + \frac{\gamma-1}{2} M^2\right) \operatorname{erfc} \eta - \frac{\gamma-1}{2} M^2 (\operatorname{erfc} \eta)^2\right]\right. \\ & \times \left[1 - \frac{\overline{\omega} a_0 T}{2 i_0}\right] + \left[\frac{2\gamma}{\sqrt{\pi}} M^2 \eta\, e^{-\eta^2} \operatorname{erf} \eta + \left(\frac{i_w}{i_0} - 1 - \frac{\gamma-1}{2} M^2\right) \frac{2\gamma}{\gamma-1} \frac{1}{\sqrt{\pi}} \eta\, e^{-\eta^2}\right] \\ & \times \left[1 - \frac{\overline{\omega} a_0 T}{6 i_0}\right] + (\gamma-1) \sqrt{\frac{2}{\pi}} M^2 e^{-\eta^2} \int_0^\eta e^{\eta^2} \operatorname{erfc} \sqrt{2}\, \eta\, d\eta \\ & + \frac{\overline{\omega} a_0 T}{i_0} \left[\sqrt{2\pi}\, \eta \operatorname{erfc} \sqrt{2}\, \eta \frac{\gamma-1}{\pi} M^2 - \sqrt{\pi}\, \eta \operatorname{erfc} \eta \left(\frac{i_w}{2 i_0} + \frac{\gamma-1}{\pi} M^2\right)\right. \\ & \left.\left. + \frac{\gamma-1}{\pi} M^2 e^{-\eta^2} (1 - e^{-\eta^2})\right]\right\}. \end{aligned} \tag{20}$$

Both the velocity correction, (19), and the enthalpy correction, (20), vanish at the wall. Far from the wall the condition is satisfied that inner and outer flow match.

These corrections, as pointed out earlier, are of the same order as the external gradient terms in equations (8) and (10). They are expressed in terms of the density-transformed co-ordinate Y, since $\eta = \frac{1}{2} Y/(\nu_0 T)^{\frac{1}{2}}$. Since the results must eventually be expressed in terms of the physical co-ordinate y, it is necessary to correct the transformation equation (11) as well.

Let $\rho = \rho_1 + \Delta\rho$, where $\rho_1 = \rho_0 i_0/i_1$ and $\Delta\rho$ is a small correction. Making use of the equation of state we can write

$$y = \int_0^Y \frac{\rho_0}{\rho} dY = y_1 + \Delta y,$$

where y_1 satisfies (11) and

$$\Delta y = \int_0^Y \left(\frac{\Delta i}{i_0} - \frac{\Delta p}{p_0}\right) dY.$$

$\Delta i/i_0$ is given by (20), and $\Delta p/p_0$ is the pressure disturbance from (18). Evaluation of the integral finally leads to

$$\frac{\Delta y}{Y} = \left[\frac{i_w}{i_0} - 1 + \frac{\gamma-1}{2} M^2(\sqrt{2}-1)\right](\gamma-1) M \left(\frac{\nu_0}{\pi U_0^2 T}\right)^{\frac{1}{2}}$$
$$\times \left\{\left[-\frac{1}{\gamma-1} - \frac{\gamma-1}{2} M^2 (\operatorname{erfc}\eta)^2 + \left(\frac{i_w}{i_0} - 1 + \frac{\gamma-1}{2} M^2\right)\operatorname{erfc}\eta\right]\left[1 - \frac{\bar{\omega} a_0 T}{2i_0}\right]\right.$$
$$+ \frac{i_w}{i_0}\frac{\bar{\omega} a_0 T}{4i_0} e^{-\eta^2} - \left[\frac{i_w}{i_0} - \frac{\bar{\omega} a_0 T}{4i_0}\left(\frac{i_w}{i_0} + \frac{2}{\pi}(\gamma-1) M^2\right)\right]\frac{\sqrt{\pi}}{2}\frac{\operatorname{erf}\eta}{\eta}$$
$$+ \left[\left(\frac{i_w}{i_0} - 1 - \frac{\gamma-1}{2} M^2\right)\left(\frac{2\gamma-1}{\gamma-1}\right) - \frac{\bar{\omega} a_0 T}{6i_0}\left(\frac{4\gamma-3}{\gamma-1}\left(\frac{i_w}{i_0} - 1\right) + \frac{2\gamma-3}{\gamma-1}\left(\frac{\gamma-1}{2} M^2\right)\right)\right]$$
$$\times \frac{1-e^{-\eta^2}}{\sqrt{\pi}\,\eta} - \left[(2\gamma-1) - \frac{\bar{\omega} a_0 T}{6i_0}(4\gamma-3)\right]\frac{M^2}{\sqrt{\pi}}\frac{e^{-\eta^2}\operatorname{erf}\eta}{\eta} + \left[(2\gamma-1) - \frac{\bar{\omega} a_0 T}{12 i_0}(11\gamma-9)\right]$$
$$\times \frac{M^2}{\sqrt{\pi}}\frac{\operatorname{erf}\sqrt{2}\,\eta}{\sqrt{2}\,\eta} + \frac{(\gamma-1) M^2}{\sqrt{2}}\left[\frac{\operatorname{erf}\eta}{\eta}\int_0^\eta e^{\eta^2}\operatorname{erfc}\sqrt{2}\,\eta\, d\eta - \frac{1}{\eta}\int_0^\eta e^{\eta^2}\operatorname{erfc}\sqrt{2}\,\eta \operatorname{erf}\eta\, d\eta\right]$$
$$+ \frac{\bar{\omega} a_0 T}{2i_0}\left[\sqrt{2\pi}\,\eta\operatorname{erfc}\sqrt{2}\,\eta\,\frac{\gamma-1}{\pi} M^2 - \sqrt{\pi}\,\eta\operatorname{erfc}\eta\left(\frac{i_w}{2i_0} + \frac{\gamma-1}{\pi} M^2\right)\right.$$
$$\left.\left. + \frac{\gamma-1}{\pi} M^2 e^{-\eta^2}(1-e^{-\eta^2})\right]\right\}. \quad (21)$$

It remains to correct the vertical velocity, (12). This is again obtained from $v = \partial y/\partial T$. Making use of (21), which together with (11) now gives y correctly to the required order, the correction is easily obtained. For simplicity, we give only the vertical velocity correction at the boundary-layer edge. This additional term, to be added to (13), is

$$\left(\frac{\Delta v}{U_0}\right)_{\eta\to\infty} = \left[\frac{i_w}{i_0} - 1 + \frac{\gamma-1}{2} M^2(\sqrt{2}-1)\right]$$
$$\times \left\{\frac{\nu_0}{U_0^2 T} M \frac{\eta}{\sqrt{\pi}}\left(1 + \frac{\bar{\omega} a_0 T}{2i_0}\right) + \frac{(\gamma-1)}{12}\frac{\nu_0 \bar{\omega}}{U_0 i_0}\left[3\frac{i_w}{i_0} + \frac{6}{\pi}(\gamma-1) M^2\right.\right.$$
$$\left.\left. - (11\gamma-9)\frac{\sqrt{2}}{\pi} M^2 - \frac{4}{\pi}\left(\frac{4\gamma-3}{\gamma-1}\right)\left(\frac{i_w}{i_0} - 1 - \frac{\gamma-1}{2} M^2\right)\right]\right\}. \quad (22)$$

This completes the second approximation to the inner flow for constant wall enthalpy.

6. Insulated wall

The analysis so far has been entirely concerned with the case of constant wall enthalpy i_w. We consider in this section the consequences of insulating the wall.

For this case it is required that $(\partial i/\partial Y)_w = 0$, and it is easily verified that this is satisfied for the first three terms on the right of (10) if

$$i_w/i_0 = 1 + \tfrac{1}{2}(\gamma-1) M^2.$$

This is then the first-order adiabatic wall temperature. Some difficulty is encountered, however, with the second-order terms due to external gradients in

(10) and in the displacement terms from (20). These terms are found to contribute to wall heat transfer, and this can be turned off only by adding an appropriate solution of the homogeneous heat conduction equation for Δi. When this is done, it is found that the part of the displacement term which remains in a uniform external flow decays only as $1/\eta$ for large η. Since boundary-layer effects should decay exponentially, another solution is required which, when added, cancels this slow decay. This solution contains $\ln(U^2T/\nu_0)$. After considerable manipulation there finally results, for velocity and enthalpy distributions,

$$\frac{u}{U_0} = \operatorname{erfc}\eta + \frac{\omega Y}{U_0} + \gamma(\gamma-1)M^3\sqrt{\frac{2}{\pi}}\left(\frac{\nu_0}{\pi U_0^2 T}\right)^{\frac{1}{2}}\left(1-\frac{\overline{\omega} a_0 T}{6 i_0}\right)\eta\, e^{-\eta^2}, \tag{23}$$

$$\begin{aligned}\frac{i}{i_0} = {} & 1+(\gamma-1)M^2\operatorname{erfc}\eta - \frac{(\gamma-1)}{2}M^2(\operatorname{erfc}\eta)^2 \\ & -\frac{\omega Y}{U_0}(\gamma-1)M^2\frac{e^{-\eta^2}}{\sqrt{\pi}\,\eta} + \frac{\overline{\omega} Y}{i_0}\left(\operatorname{erf}\eta + \frac{e^{-\eta^2}}{\sqrt{\pi}\,\eta}\right) + \frac{(\gamma-1)^2}{\sqrt{2}}M^3\left(\frac{\nu_0}{\pi U_0^2 T}\right)^{\frac{1}{2}} \\ & \times\left\{\left[1+(\gamma-1)M^2\operatorname{erfc}\eta - \frac{\gamma-1}{2}M^2(\operatorname{erfc}\eta)^2\right]\left[1-\frac{\overline{\omega} a_0 T}{2 i_0}\right]\right. \\ & + \left[\frac{2\gamma}{\sqrt{\pi}}M^2\eta\, e^{-\eta^2}\operatorname{erf}\eta\right]\left[1-\frac{\overline{\omega} a_0 T}{6 i_0}\right] - \left(1+\frac{\gamma-1}{2}M^2\right)e^{-\eta^2} \\ & - \frac{\overline{\omega} a_0 T}{i_0}(\operatorname{erfc}\sqrt{2}\,\eta - \operatorname{erfc}\eta) - (\gamma-1)M^2\sqrt{\frac{2}{\pi}}e^{-\eta^2}\int_0^\eta e^{\eta^2}(\operatorname{erfc}\sqrt{2}\,\eta - \operatorname{erfc}\eta)\,d\eta \\ & \left. - \frac{\gamma-1}{\sqrt{2}}\frac{M^2}{\pi}e^{-\eta^2}\left[\ln\left(\frac{U_0^2 T}{\nu_0}\right) - C - \frac{\overline{\omega} a_0 T}{i_0}(2-\sqrt{2}\,e^{-\eta^2})\right]\right\}. \end{aligned} \tag{24}$$

Equation (23), when $\omega = \overline{\omega} = 0$, is identical to that obtained by Van Dyke (1952). In equation (24) it is to be noted that the external-enthalpy-gradient term not connected with displacement is no longer additive. As was the case for constant wall enthalpy, the displacement-induced term contains contributions from this gradient. Further, in that part of the solution which remains with uniform external flow only the first and fourth displacement terms of (24) appear in Van Dyke (1952). This discrepancy can be traced to the neglect by Van Dyke of first-order density variations in obtaining the inhomogeneous term in (7).

The constant C appearing in the last term of (24) cannot be determined within the framework of the solution. Its occurrence is not related to the existence of initial external gradients. Second-order displacement corrections for y and v also will be indeterminate, and they have not been calculated for the insulated case. The non-uniqueness of the solution is analogous to the situation which arises when determining higher-order corrections in the incompressible flow over a semi-infinite flat plate, as described by Van Dyke (1960). An important difference is that the logarithmic term is necessary in the present degree of approximation, whereas for the flat-plate boundary layer it does not occur until the next. It is of interest to note that in the Rayleigh problem of Hanin (1960) no logarithmic term is encountered due to his linearization. This restricts his solution to low Mach numbers such that terms equivalent to M^2-terms inside the braces of (24) are ignored, and it is just this kind of term which contains the logarithmic contribution.

7. Results and discussion

Having obtained consistent expressions for both u and i, we are now ready to compute both skin friction and heat transfer at the wall.

For the skin friction coefficient C_f write

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho_0 U_0^2} = \frac{(\mu\partial u/\partial y)_w}{\frac{1}{2}\rho_0 U_0^2} = \frac{2\nu_0}{U_0}\left(\frac{p}{p_0}\frac{\partial(u/U_0)}{\partial Y}\right)_{\eta=0},$$

where τ_w is the wall shearing stress. From (8), (18) and (19) there results

$$C_f = -2\left(\frac{\nu_0}{\pi U_0^2 T}\right)^{\frac{1}{2}} + \frac{2\nu_0\omega}{U_0^2} + \frac{2\gamma}{3\pi}\frac{\nu_0\bar{\omega}}{U_0 i_0}\left[\frac{i_w}{i_0} - 1 + \frac{\gamma-1}{2}M^2(\sqrt{2}-1)\right]. \tag{25}$$

This expression is equally valid for the heat-transfer case and for the insulated wall with first-order adiabatic wall temperature. The first term on the right in (25) is the classical constant-density result; it is uninfluenced by Mach number or wall enthalpy because of the assumption that μ is proportional to i, as also pointed out by Van Dyke. The second term is a simply additive contribution of the external shear which increases wall friction if the external shear has the same sign as that near the wall ($\omega < 0$). The third term is an interaction term which can either increase or decrease skin friction. For $i_w/i_0 = 1$ an increase results for an outwardly decreasing external enthalpy. For vanishing

$$(i_w/i_0) - 1 + \tfrac{1}{2}(\gamma-1)M^2(\sqrt{2}-1)$$

this interaction term disappears. Throughout, displacement effects for constant wall enthalpy have entered multiplied by this factor. It first appears in (13) and subsequently in all displacement-induced terms. These thus all vanish when

$$i_w/i_0 = 1 - \tfrac{1}{2}(\gamma-1)M^2(\sqrt{2}-1).$$

With this wall enthalpy and a uniform external flow, the vertical velocity at the boundary-layer edge vanishes to both first and second order. For a colder wall this induced velocity will be negative. Equation (25) is correct to order $\nu_0/U_0^2 T$. Terms involving displacement interaction in a uniform external flow are found to cancel out in the expression for skin friction and this effect does not appear.

The same procedure is followed in evaluating wall heat-transfer. The Stanton number is written as

$$C_h = \frac{q_w}{\rho_0 U_0(i_w - i_{aw})} = -\frac{(k\partial T/\partial y)_w}{\rho_0 U_0(i_w - i_{aw})} = \left(\frac{i_0}{i_{aw} - i_w}\right)\frac{\nu_0}{U_0}\left(\frac{p}{p_0}\frac{\partial(i/i_0)}{\partial Y}\right)_{\eta=0},$$

where q_w is the wall heat-transfer and i_{aw} is the adiabatic wall enthalpy. This, of course, depends also on second-order effects. In fact, from (24) evaluated at $\eta = 0$, there results

$$\frac{i_{aw}}{i_0} = 1 + \frac{\gamma-1}{2}M^2 + \frac{(\gamma-1)^3}{2\pi}M^5\left(\frac{\nu_0}{\pi U_0^2 T}\right)^{\frac{1}{2}}\left[C - \ln\left(\frac{U_0^2 T}{\nu_0}\right)\right] - \frac{2}{\pi}\frac{\omega(\pi\nu_0 T)^{\frac{1}{2}}}{U_0}(\gamma-1)M^2$$
$$+ \frac{2}{\pi}\frac{\bar{\omega}(\pi\nu_0 T)^{\frac{1}{2}}}{i_0}\left\{1 - \frac{(\gamma-1)^2 M^2}{4\sqrt{2}}\left[1 + \frac{\gamma-1}{2}M^2\left(1 - \frac{4}{\pi}(\sqrt{2}-1)\right)\right]\right\}. \tag{26}$$

As an analogue to the blunt-body boundary layer consider $\omega < 0$, $\overline{\omega} < 0$. With external shear of the same sign as the shear near the wall, the insulated wall is made hotter. With external enthalpy decreasing outward the wall is made cooler for small M, hotter for large M. The conventional displacement effect on the adiabatic wall enthalpy is indeterminate, but it can be seen that the effect enters as $M^5(\nu_0/U_0^2T)^{\frac{1}{2}}$, a surprisingly fast variation with Mach number. The corresponding parameter, as obtained by Van Dyke (1952), contains M^3. Thus the use of constant first-order density previously mentioned eventually leads to a missing factor of the square of the Mach number in the adiabatic wall enthalpy.

Since i_{aw} contains an undetermined constant it is convenient to use the first-order adiabatic wall enthalpy in the definition of C_h. Then, from (10), (20) and (18),

$$\begin{aligned}C_h = \left(\frac{\nu_0}{\pi U_0^2 T}\right)^{\frac{1}{2}} \Bigg\{ 1 &+ \frac{\omega(\pi\nu_0 T)^{\frac{1}{2}}}{U_0} \frac{(\gamma-1)M^2}{(i_w/i_0)-1-\frac{1}{2}(\gamma-1)M^2} + \frac{\overline{\omega}(\pi\nu_0 T)^{\frac{1}{2}}}{i_0} \\ &\times \left[\frac{\gamma-1}{4}\left(\frac{i_w}{i_0}-1+\frac{\gamma-1}{2}M^2(\sqrt{2}-1)\right)\left(1+\frac{(6-\sqrt{2})(\gamma-1)+2}{3\pi(\gamma-1)}(\gamma-1)M^2\right.\right. \\ &\left.\left. - \frac{(6-3\pi)(\gamma-1)+4\gamma}{3\pi(\gamma-1)}\left\{\frac{i_w}{i_0}-1\right\}\right)-1\right]\left[\frac{i_w}{i_0}-1-\frac{\gamma-1}{2}M^2\right]^{-1} + (\gamma-1)M\left(\frac{\nu_0}{\pi U_0^2 T}\right)^{\frac{1}{2}} \\ &\times \left[\frac{i_w}{i_0}-1+\frac{\gamma-1}{2}M^2(\sqrt{2}-1)\right]\frac{(i_w/i_0)-1-\frac{1}{2}(\gamma-1)M^2(\sqrt{2}+1)}{(i_w/i_0)-1-\frac{1}{2}(\gamma-1)M^2}\Bigg\}. \qquad (27)\end{aligned}$$

The last term in the numerator of the expression involving the initial enthalpy gradient is a simply additive contribution from the initial heat transfer. For $i_w/i_0 = 1$,

$$\begin{aligned}C_h = \left(\frac{\nu_0}{\pi U_0^2 T}\right)^{\frac{1}{2}} \Bigg\{ 1 &+ \frac{(\gamma-1)^2}{2} M^3 \left(\frac{\nu_0}{\pi U_0^2 T}\right)^{\frac{1}{2}} - \frac{2\omega(\pi\nu_0 T)^{\frac{1}{2}}}{U_0} \\ &- \frac{2\overline{\omega}(\pi\nu_0 T)^{\frac{1}{2}}}{i_0}\left[\frac{\gamma-1}{8}(\sqrt{2}-1)\left(1+\frac{(6-\sqrt{2})(\gamma-1)+2}{3\pi}M^2\right)-\frac{1}{(\gamma-1)M^2}\right]\Bigg\}.\end{aligned}$$

If the wall enthalpy is maintained at the first-order adiabatic wall enthalpy or less, then with negative external gradients the velocity gradient increases heat transfer. Heat transfer is increased by the enthalpy gradient at large M and decreased for small M. The conventional displacement effect can either increase or decrease wall heat transfer. For $i_w/i_0 = 1$ an increase results. For

$$i_w/i_0 = 1-\tfrac{1}{2}(\gamma-1)M^2(\pm\sqrt{2}-1),$$

there is no contribution at all. It can also be readily shown from (25) and (27) that considerable reductions in both vorticity and conventional displacement interaction occur for a cold wall ($i_w/i_0 = 0$) compared to a hot wall

$$(i_w/i_0 = 1+\tfrac{1}{2}(\gamma-1)M^2).$$

We may at this point compare results with steady, two-dimensional flow over a semi-infinite flat plate. In the latter case weak interaction theory with uniform external flow shows that the effect of boundary-layer displacement increases

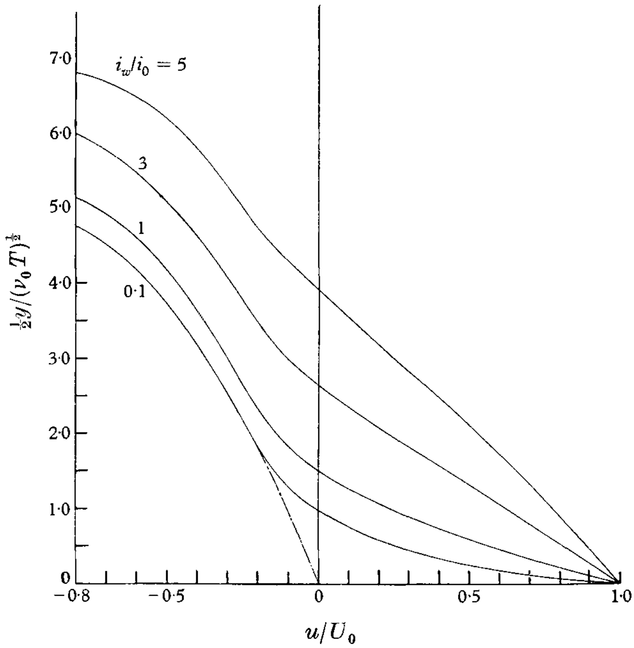

FIGURE 2. Velocity profiles for various wall enthalpies, fixed time, $M = 3$, $(\nu_0/U_0^2 T)^{\frac{1}{2}} = 0{\cdot}01$; —·—, initial profile.

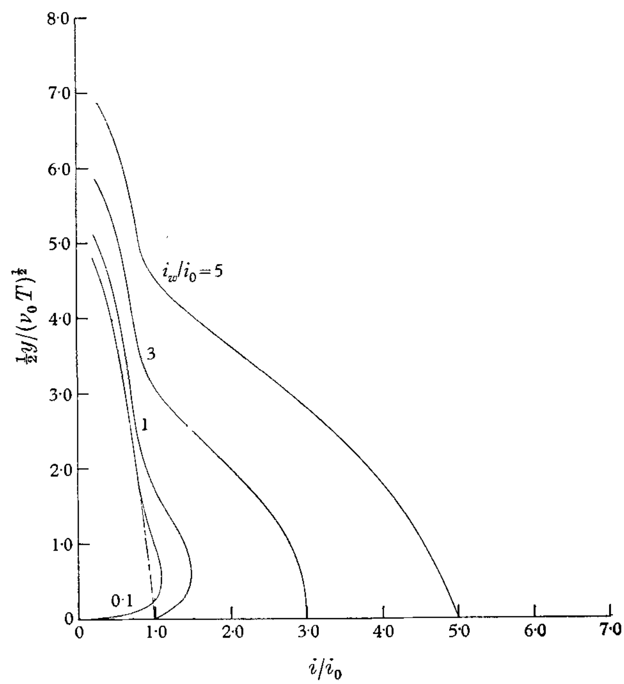

FIGURE 3. Enthalpy profiles for various wall enthalpies, fixed time, $M = 3$, $(\nu_0/U_0^2 T)^{\frac{1}{2}} = 0{\cdot}01$; —·—, initial profile.

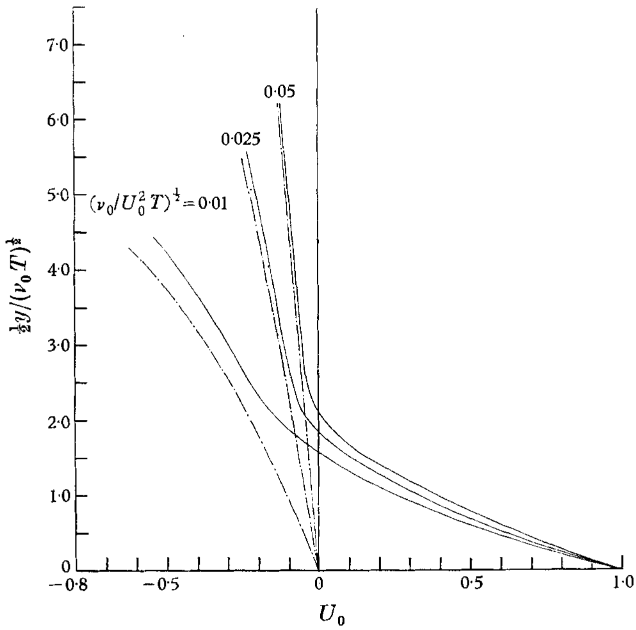

FIGURE 4. Velocity profiles for various times, fixed wall enthalpy, $M = 3$, $i_w/i_0 = 1$; —·—, initial profile.

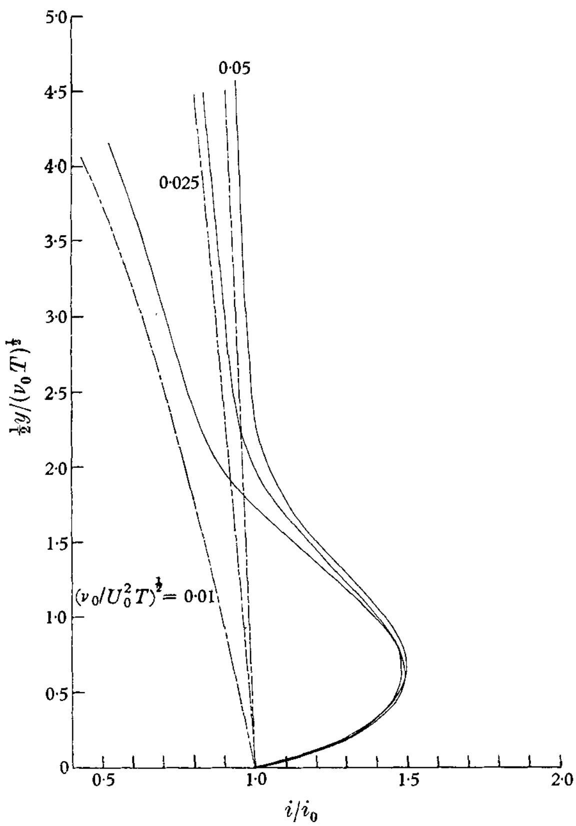

FIGURE 5. Enthalpy profiles for various times, fixed wall enthalpy, $M = 3$, $i_w/i_0 = 1$; —·—, initial profile.

skin friction but does not influence heat transfer. This is just the opposite from what is found here with uniform flow (see equations (25) and (27) with $\omega = \overline{\omega} = 0$). Thus higher-order effects can be quite different in various apparently related cases. This has also just been seen in the nature of the conventional displacement effect for contant wall enthalpy and the insulated Rayleigh plate. Comparison with the effects of the external gradients obtained by Maslen (1962) for the flat plate again shows different and much more complex behaviour in the latter case.

Finally, some velocity and enthalpy profiles are presented as a function of the normalized physical co-ordinates $y/2(\nu_0 T)^{\frac{1}{2}}$. For the insulated wall, the indeterminacy in the solution prevents display of any profiles. For the heat-transfer case, $M = 3$ and $\nu_0 \omega/U_0^2 = \nu_0 \overline{\omega}/U_0 i_0 = -5 \times 10^{-4}$ have been selected. For illustrative purposes figures 2 and 3 show velocities and enthalpies for various values of i_w/i_0 for $(\nu_0/U_0^2 T)^{\frac{1}{2}} = 1 \times 10^{-2}$. Figures 4 and 5 indicate these profiles for $i_w/i_0 = 1{\cdot}0$ with various values of $(\nu_0/U_0^2 T)^{\frac{1}{2}}$. In all cases the initial profiles are also indicated.

8. Conclusions

The impulsively moved compressible Rayleigh plate in a non-uniform initial flow has been considered for two cases, that of constant wall enthalpy with heat transfer and that of an insulated wall.

It is found that skin friction contains a simply additive contribution from the initial shear and wall heat transfer contains a simply additive contribution from initial heat transfer. However, displacement-induced effects are influenced by the initial enthalpy gradient and this leads to interaction terms which affect skin friction, heat transfer and adiabatic wall enthalpy. These terms can either increase or decrease skin friction and wall heat-transfer because the induced velocity normal to the plate can be either positive or negative depending on surface enthalpy and Mach number. If the initial heat transfer is away from the wall, skin friction is increased for the insulated plate. Adiabatic wall enthalpy then decreases due to initial enthalpy gradient at low Mach number and increases at high Mach number. It is increased by initial shear if this is of the same sign as wall shear.

That part of the displacement-induced effect which remains with uniform initial flow affects heat transfer but not skin friction to the order considered. This is just the opposite of the case of weak interaction for a two-dimensional semi-infinite flat plate. This conventional displacement can also either increase or decrease wall heat-transfer.

The insulated case and the constant wall enthalpy case differ principally in that a logarithmic term in Reynolds number (based on time) enters into conventional displacement for the insulated problem but not for the other. This term is accompanied by an undetermined constant which makes the adiabatic wall enthalpy indeterminate.

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